ELECTRONIC FINANCIAL NETWORKS WITH SOCIALLY RESPONSIBLE INVESTING

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Abstract

Advances in telecommunication networks, and, in particular, the Internet, have transformed the economic landscape for financial decision-making. Meanwhile the past and current financial crisis calls for social responsibility which is considered by many stack-holders as a potential solution to avoid future crisis. In this paper, we develop a framework for the modeling, analysis, and computation of solutions to multi-tiered financial network problems with electronic transactions and socially responsible investment in which both the sources of financial funds as well as the intermediaries are multi-criteria decision-makers. We assume that these decision-makers seek not only to maximize their net revenues but also minimize risk with the risk being penalized by a variable weight. Furthermore, we assume that the intermediaries are socially responsible companies, which want to maximize their social responsibility. We make explicit the behavior of the various decision-makers, including the consumers at the demand markets for the financial products. We derive the optimality conditions, and demonstrate that the governing equilibrium conditions of the financial network economy can be formulated as a finite-dimensional variational inequality problem. Qualitative properties of the equilibrium financial flow and price pattern are also provided.

Keywords: *Socially responsible investing, electronic transaction, financial networks, variational inequality, variable weights*

INTRODUCTION

Advances in telecommunications and, in particular, the adoption of the Internet by businesses, consumers, and financial institutions have had an enormous effect on financial services and the options available for financial transactions. Distribution channels have been transformed, new types of services and products introduced, and the role of financial intermediaries altered in the new economic networked landscape. Furthermore, the impact of such advances has not been limited to individual nations but, rather, through new linkages, has crossed national boundaries.

The topic of *electronic* finance has been a growing area of study (cf. Claessens, Glaessner and Klingebiel, 2000; 2001; Long, 2000; Sato and Hawkins, 2001; Banks, 2001; and Allen, Hawkins and Sato, 2001, and the references therein), due to its increasing impact on financial markets and financial intermediation, as well as related regulatory issues and governance (see also Turner, 2001). Of particular emphasis has been the conceptualization of the major issues involved and the role of networks in the transformations (see McAndrews and Stefanidis, 2000; Allen et al., 2001; Economides, 2001;Nagurney and Dong, 2002;Nagurney and Ke, 2003).

Nevertheless, the complexity of the interactions among the distinct decision-makers involved, the supply chain aspects of the financial product accessibilities and deliveries, as well as the availability of physical and electronic options, and the role of intermediaries, have defied the construction of a unified, quantifiable framework in which one can assess the resulting financial flows and prices.

In the meantime, socially responsible investing (SRI) funds have been an important strategy for many investors who believe in "doing good while doing well". SRI is defined as an investment process that considers environmental, social, and governance consequences of investments, both positive and negative, within the context of financial analysis (EUROSIF, 2012). Bilbao-Terol, Arenas-Parra, Cañal-Fernández and Bilbao-Terol (2015) point out that "Ethicality in financial business has never been as necessary as today because it is generally recognized that unethical behaviors conducted in recent years have led to the current crisis." SRI combines investors' financial objectives with their concerns about social, environmental and ethical (SEE) issues (Social Investment Forum). This investment concept has gained much more visibilities among the investors around the world recently (Jun, 2016; Ang, Gregoriou, and Lean, 2014). Ethical or socially responsible investing advocates the practices such as sustainable development, the advancement of human rights, and consumer protection. Although the first sustainable index, the Domini 400 Social Index, was launched only in 1990 (Guerard, 1997), the number of sustainable indices has grown significantly in the past few years due to the widely acceptance of SRI within the investment communities. The Experts in Responsible Investment Solutions (EIRIS) reports the number of green and ethical funds has increased from 24 in year of 2000 to almost 100 in 2010. There are £15 billion invested in UK green and ethical retail funds, increased from £1.5 billion 15 years ago (www.eiris.org, 2015). Many organizations and rating agencies have offered such indices, which for example, include Dow Jones Sustainability Indices (DJSI), E.Capital, FTSE4, Humanix, ESG by Vigeo, etc.

Screening and shareholder advocacy are two main ways of applying SRI principles in the investment practice (see Boutin-Dufresne and Savaria, 2004 and Stephen and Hope, 2007; Humphrey, 2015). Negative screening excludes certain securities from investment portfolio based on social and/or environmental criteria. For example, companies that are involved in tobacco, nuclear, and weapon are avoided by many SRI funds (Statman, 2006). Plihon and Ponssard (2002) documented that early SRI were motivated by a desire to exclude certain firms from the portfolios that engaged in businesses not aligned with their religious beliefs. To facilitate the screening process, various socially responsible ratings are provided by many research institutes. For example, after conducting a survey among consumers regarding to the sustainable practice of the 230 largest and most visible U.S. companies, the Reputation Institute and the Center for Corporate Citizenship at Boston College published a report that ranks the corporate social responsibility of these firms. The ranking score ranges from 0 to 100, with 0 being the least social responsible and 100 being the most social responsible (Reputation Institution, 2010). There are other websites that publish the similar ratings as well, such as: www.csrhub.com, corpwatch.org, and www.sustainability-indexes.com, just to name a few. These ratings help investors select and screen their portfolio based on the SRI criteria more effectively. On the other hand, positive screening, unlike the negative screening, which merely excludes companies with socially irresponsible products, takes a more proactive investing approach: it systematically incorporates the corporate social responsibility factors into the process of portfolio selection and investment. Examples of positive screens are progressive hiring policies, development of environmentally friendly technologies or involvement with community development (Humphrey, 2015).The positive screening has been evidently seen in practice. In a survey conducted in 1997, it has been reported that 81 percent of the investors favored the incorporation of sustainable factors into their investment decisions when choosing fund managers (Krumsiek,1997).

Shareholder advocacy, being another important aspect of SRI practice, is to influence the corporate behavior by encouraging social investors to work cooperatively to steer companies on a socially sustainable course. More companies have started to act on such a demand from investors. For example, it is reported that of the 250 largest multinational corporations, 64 percent published corporate social responsibility reports in 2005, either within their annual report or in separate sustainability reports (Porter and Kramer, 2006).

Humphrey (2015) noticed that how SRI are screened varies substantially from country to country. For example, most SRI funds in Australia use environmental screening while shareholder advocacy is confined to the US market. In France, the majority of SRI products use best-in-class methodology while in the Netherlands almost no SRI products use this methodology.

Besides ethical reasons, a better than average performance of the SRI related stocks contributes to another reason why SRI has been more popular recently. According to a recent report issued by Social Investment Forum Foundation

(2010), despite the recent economic downturn, SRI assets in the United States has increased by 34 percent while the overall professionally managed assets have increased only 3 percent since 2005. Furthermore, during the period from 2007 to 2009, the assets associated with SRI have amounted to $3.07 trillion, which represents more than 13 percent comparing to the less than 1 percent increase of the broad market. In addition, the European SRI market grew from €1 trillion in 2005 to €6.76 trillion in 2011 (European Sustainable Investment Forum). Research has found that SRI founds actually performed equally with traditional standards market benchmarks as the S&P 500 indicating that SRI funds can be considered as alternative investment for investors who would like to contribute to society (Ang et al., 2014).

A French study found negative screening decreased returns and a US study found positive screening increased returns. Humphrey (2015) argues that positive screening may increase returns and decrease risk due to firms' good relationships with their stakeholders. In contrast, the negative screening should decrease returns and increase risk a highly profitable sin stocks are excluded from the portfolio and fully diversified portfolios can't be formed. Auer (2016) finds out that negative screens excluding unrated stocks from a representative European stock universe allow investors to significantly outperform a passive investment, additional negative screens based on environmental and social scores neither add nor destroy portfolio value, indeed governance screens can significantly increase portfolio performance, however positive screens can cause portfolios to underperform the benchmark due to loss of diversification.

The research on SRI is mostly focused on using empirical data to compare the performance of SRI funds with the conventional benchmark indices and funds. For example, Hickman, Teets, and Kohls (1999) found that portfolio risks are reduced by including social responsible companies even though the investors are not driven by the social values. Van de Velde, Vermeir, and Corten (2005) used the Vigeo corporate responsibility scores (VCRS) to construct four different portfolios ranging from best to worst with regard to their VCRS. The authors then compared the performance among these portfolios and found that portfolios with high VCRS have performed better than the ones with low VCRS. Furthermore, Pava and Krausz (1996) and Preston and O'Bannon (1997) indicated that companies'financial performance was improved by their corporate responsibility levels. Other researchers showed that the corporate social enhanced the companies' relationship with shareholders and therefore, created synergies. (Stanwick and Stanwick,1998; Verschoor,1998). On the other hand, there are researchers who found negative links between the social responsibility levels and the performance of the funds. (Géczy, Stambaugh, and Levin, 2003; Kurtz,1997; McWilliams and Siegel,1997). However, as noted by Statman (2006), the reason for the above could be that the social investors overly valuated the price of the socially responsible company.

Renneboog, TerHorst, and Zhang (2011) studied the money flows into and out of SRI funds around the world and found out that stock picking based on in-house SRI search increases the money flows. Basso and Funari (2014) also suggest that the ethical objective can be pursued without having to renounce financial rewards.

Although the amount of research on SRI has been increasing significantly in the past decade, the majority of them are focused on either qualitative or empirical research. To the best of our knowledge, there is no quantitative model that captures the social awareness of the investing companies especially considering the impact of electronic transaction together with other important objectives, such as risk minimization. Therefore, we believe that such a general theoretical model is timely and relevant. Moreover, the model can also provide insights and directions for the future empirical research.

We also believe that to fully understand the complexity of nowadays financial networks, it is imperative to study the interactions among different decision-makers in a *multi-tiered network* setting. Within this network, investors/financial institutes compete within a tier but in order for financial products to be eventually delivered to the consumers, the entities between different tiers must cooperate at the same time. In addition, since our model captures the complete financial network and its resulting equilibrium state, it provides a valuable reference to analyze the existing prices and financial flows. For a comprehensive review on network optimization for financial engineering problems, see Guenes and Pardalos (2003). For a review on financial networks, see Nagurney (2008). For reviews on the time-dependent financial networks and the resulting network equilibrium, see Daniele, Giuffre and Pia (2005) and Giuffré and Pia (2005).For a review on multiple criteria decision-making applied to problems and issues in finance, see the work by Steuer and Na (2003) and Ballestero, Pérez-Gladish, and Garcia-Bernabeu (2015).

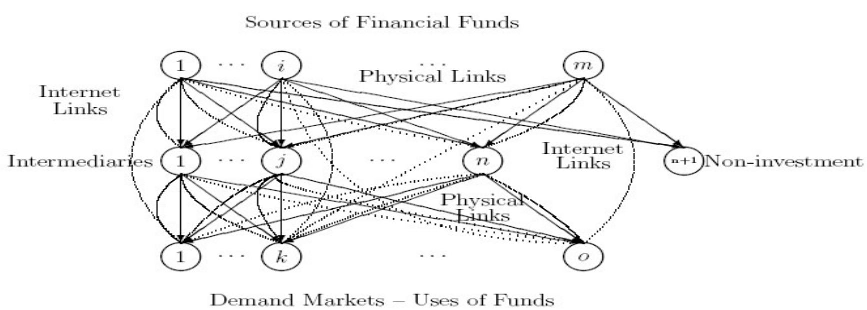
The rest of the paper is organized as follows: in Section 2, the electronic financial network model with socially responsible intermediaries and variable weights are introduced and the financial network equilibrium conditions are defined. Qualitative properties of the equilibrium solutions are discussed in Section 3. An algorithm is introduced in Section 4. Finally, the paper is concluded in Section 5 with summary and future research directions.

THE FINANCIAL NETWORK MODEL WITH ELECTRONIC TRANSACTIONS, SOCIAL RESPONSIBILITY INVESTING, AND VARIABLE WEIGHTS

In this Section, we develop the financial network model consisting of: agents or decision makers with sources of funds, financial intermediaries, as well as consumers associated with the demand markets. In this model, the sources of funds can conduct their financial transactions with the intermediaries either physically or electronically. The intermediaries, in turn, transact with the consumers. The depiction of the network at equilibrium is given in Figure 1.

Specifically, we consider m agents with sources of financial funds, such as households and businesses, involved in the allocation of their financial resources among a portfolio of financial instruments which can be obtained by transacting with n distinct financial intermediaries, such as banks, insurance and investment companies, social responsibility investing institute, etc. The financial intermediaries, in turn, in addition to transacting with the source agents, determine how to allocate the incoming financial resources among distinct uses or financial products at the demand markets, such as, for example, the market for real estate loans, household loans, or business loans, etc. The financial network is now described and depicted graphically in Figure 1. The top tier of nodes consists of the decision-makers with sources of funds, with a typical source agent denoted by i and associated with node i. The middle tier of nodes consists of the intermediaries, with a typical intermediary denoted by j and associated with node j in the network. In addition, in contrast to the model of Nagurney and Ke (2001; 2003), we now allow for the possibility of source agents not investing their funds (or a portion thereof), which we represent by the node n + 1 at the middle tier of nodes. The intermediaries are assumed to receive financial returns from the demand market. For example, an intermediary may use the fund from the source agents to buy the stock of a publicly traded company's stock. In return, this publicly traded company will pay the intermediary a financial return, such as dividends. Furthermore, the intermediaries, as socially responsible investing companies, will encourage corporate practices that promote environmental stewardship, consumer protection, human rights, and diversity. They avoid businesses involved in alcohol, tobacco, gambling, pornography, weapons, and/or the military. The bottom tier of nodes consists of the demand markets, with a typical demand market denoted by k and corresponding to node k (and associated with a particular financial product).

FIGURE 1
The Structure of the Financial Network with Electronic Transactions



We now describe the behavior of the various economic decision-makers represented by the three tiers of nodes in Figure 1. We first focus on the source agents. We then turn to the intermediaries and, subsequently, to the demand markets. To make the presentation clear, we also list the relevant variables/notations below:

- q_{ijl}^{1}isnonnegative amount of the funds that source i "invests" in financial instrument j obtained from intermediary j through transaction mode l where l=1 is physical transaction and l=2 represents electronic transaction. Group the financial flows of a source agent i, which are associated with the links emanating from top-tier node i, into the column vector $q_i^1$and group the financial flows of all such source agents into the mnL-dimensional vector $Q^1 \in R_+^{2mn}$.

- q_{ik}^1 is the amount of transaction between source agent i and demand market k through electronic transaction. Group these variables into a column vector $Q^2 \in R_+^{mo}$.
- q_i^1 is a column vector of 2n+o， including all transactions of source agent i.
- q_{jkl}^2 is amount of the financial product obtained by consumers at demand market k from intermediary j through transaction mode l where l=1 is physical transaction and l=2 represents electronic transaction via Internet. Group these "consumption" quantities into the column vector $Q^3 \in R_+^{2no}$. The intermediaries, in turn, convert the incoming financial flows Q^1 into the outgoing financial flows Q^3.
- c_{ijl} is the transaction cost between source agent i and intermediary j through transaction mode l.
- c_{ik} is the transaction cost between source agent i and demand market k.
- c_{jkl} is the transaction cost between intermediary j and demand market k through transaction mode l.
- ρ is the related price of the financial product.
- S_i is the amount of funds that source agent i has.
- c_j is the handling cost of intermediary j.
- $\hat{c}_{ijl}$ is the transaction costwhen intermediary j transacts with source agent i through transaction mode l.
- I_k is the social responsibility level of the financial product on demand market k. The value of I_k ranges from 0 to 1, with 0 being the least social responsible and 1 being the most social responsible.
- g_j is the minimum social responsibility level that intermediary j specifies after investing in the financial products.

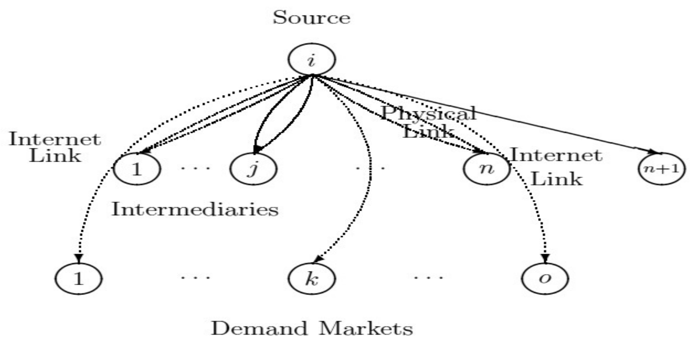
Please refer to Nagurney and Ke (2003; 2006) and Qiang, Ke and Hu (2014) for more specific definitions.

The Behavior of the Agents with Sources of Funds and their Optimality Conditions

For each source agent i, the amount of funds allocated by him cannot exceed his financial holdings. Therefore, the following conservation of flow equation must hold:

$$\sum_{j=1}^{n} \sum_{l=1}^{2} q_{ijl}^1 + \sum_{k=1}^{o} q_{ik}^1 \le S_i, \forall i. \quad (1)$$

FIGURE 2
Network Structure of Source Agent i's Transactions



Assume that each source agent faces a multicriteria decision-making problem, with the first objective reflecting net revenue maximization and expressed as:

$$\text{Maximize} \quad z_{1i}^{1} = \sum_{j=1}^{n}\sum_{l=1}^{2}\rho_{ijl}^{1}q_{ijl}^{1} + \sum_{k=1}^{o}\rho_{ik}^{1}q_{ik}^{1} - \sum_{j=1}^{n}\sum_{l=1}^{2}c_{ijl}(q_{ijl}^{1}) - \sum_{k=1}^{o}c_{ik}(q_{ik}^{1}) \tag{2}$$

and the second objective denoting risk minimization and expressed as:

$$\text{Minimize} \quad z_{2i}^{1} = r^{i}(q_{i}^{1}) \tag{3}$$

Furthermore, a source agent is faced with trading off the gain of one objective against the other objective. The essence of the issue is, "How much achievement on objective z_{1i}^{1} is the decision-maker willing to give up in order to improve achievement on objective z_{2i}^{1} by some amount?" It is rational to assume that financial investors are risk-averse and for them, the weights of the two objectives may not be equal. For example, a risk-averse investor may be willing to accept a portfolio with a lower mean return if the portfolio has a lower associated risk. In other words, the risk-averse investor may be willing to take on certain risks only if the return is much higher. In general, individual agents may have limited funds to make investment, which makes it more difficult for them to diversify among different securities in end market through direct investment. Therefore, their ability to diversify risks would be weaker than financial intermediaries which can gather large amount of funds from individual investors and construct diversified portfolios. As discussed in Dong and Nagurney (2001), more attention will generally be given to reduce the risk when the risk is high and this kind of decision rationality argues that the objective function should penalize the states with high risk by imposing a greater weight to z_{2i}^{1}associated with high risks than to those z_{2i}^{1} with low risks. We adopt the same definition of Criterion-Dependent Weight defined in Nagurney and Ke (2006), we then derive the risk-penalizing value function as follows (Nagurney and Ke, 2006; Dong and Nagurney, 2001):

Maximize $\mathrm{U^i}\left(\mathrm{q_i^1}\right) = \sum_{j=1}^{n}\sum_{l=1}^{2}\left(\rho_{ijl}^1 q_{ijl}^1 - c_{ijl}\left(q_{ijl}^1\right)\right) + \sum_{k=1}^{o}(\rho_{ik}^1 q_{ik}^1 - c_{ik}(q_{ik}^1)) - w_{2i}^1\left(r^i\left(q_i^1\right)\right) r^i\left(q_i^1\right)$ (4)

and the constraint (1) for this agent i.

The expression consisting of the first four terms to the right-hand side of the equal sign in (4) represents the net revenue (which is to be maximized), whereas the last term in (4) represents the weighted dollar value of risk (which is to be minimized) by source agent i, where $w_{2i}^1(\mathrm{r^i}\left(\mathrm{q_i^1}\right))$ denotes the risk-penalizing weight associated with the value of the risk objective of source agent i. Furthermore, w_{2i}^1 is assumed to be a strictly increasing, convex, smooth, and nonnegative function (Nagurney and Ke, 2006; Dong and Nagurney, 2001). It is also possible that the weight function can be affected by external factors such as macroeconomic condition and investors' sentiment, given the risks faced by agent i.Observe that such an objective function is in concert with those used in classical portfolio optimization (see Markowitz, 1952). Note that value functions have been studied extensively and used for decision problems with multiple criteria (cf. Fishburn, 1970; Zeleny, 1982; Chankong and Haimes, 1983; Yu, 1985; and Keeney and Raiffa, 1993). Of course, a special example of a constant weight value function is the one with equal weights (see, e.g., Nagurney and Ke, 2003). For some references to the use of value functions with equal weights that have been used in financial applications, see Dong and Nagurney (2001).

Analogue to the proof of Theorem 1 in Nagurney and Ke (2006) and Dong and Nagurney (2001), we can prove that (4) is strictly concave with respect to $\mathrm{q_i^1}$. We then can derive the optimality conditions of (5) (Bazaraa, Sherali, and Shetty, 1993):

$$-\sum_{j=1}^{n}\sum_{l=1}^{2}\frac{\partial U^i(q_i^{1*})}{\partial q_{ijl}^1}\times\left(q_{ijl}^1 - q_{ijl}^{1*}\right) - \sum_{k=1}^{o}\frac{\partial U^i(q_i^{1*})}{\partial q_{ik}^1}\times\left(q_{ik}^1 - q_{ik}^{1*}\right) \geq 0 \qquad (5)$$

Therefore, the optimality conditions for all source agents *simultaneously* can be expressed as the following inequality (see also Nagurney and Ke, 2001): determine $\mathrm{Q^{1*}} \in K_1$, such that:

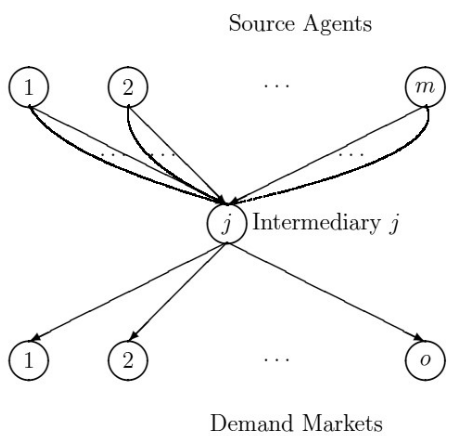
$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[w_{2i}^1\left(r^i\left(q_i^{1*}\right)\right)\frac{\partial r^i(q_i^{1*})}{\partial q_{ijl}^1} + \frac{\partial w_{2i}^1\left(r^i(q_i^{1*})\right)}{\partial q_{ijl}^1} r^i\left(q_i^{1*}\right) + \frac{\partial c_{ijl}\left(q_{ijl}^{1*}\right)}{\partial q_{ijl}^1} - \rho_{ijl}^{1*}\right]\times\left[q_{ijl}^1 - q_{ijl}^{1*}\right] + \sum_{i=1}^{m}\sum_{k=1}^{o}\left[w_{2i}^1\left(r^i\left(q_i^{1*}\right)\right)\frac{\partial r^i(q_i^{1*})}{\partial q_{ik}^1} + \frac{\partial w_{2i}^1\left(r^i(q_i^{1*})\right)}{\partial q_{ik}^1} r^i\left(q_i^{1*}\right) + \frac{\partial c_{ik}(q_{ik}^{1*})}{\partial q_{ik}^1} - \rho_{ik}^{1*}\right]\times\left[q_{ik}^1 - q_{ik}^{1*}\right] \geq 0, \forall (Q^1, Q^2) \in \mathrm{K1} \qquad (6)$$

Where $\mathrm{K1} \equiv \left\{(Q^1, Q^2)\middle| q_{ijl}^1 \geq 0, \forall i, j, l; q_{ik}^1 \geq 0, \forall i, k, \text{and } (1)\text{holds}\right\}$

The Behavior of the Intermediaries and their Optimality Conditions

We now describe the behavior of the financial intermediaries. For a graphical depiction of the transactions associated with intermediary j, see Figure 3. As discussed in Introduction, we assume that intermediaries are socially responsible investors. They invest the money obtained from the source agents into the financial products, which are evaluated on their social responsibility level by the rating agencies. Hence, the social responsibility level of an intermediary is reflected by the allocation of the percentage of the money that he/she invested in all the financial products. If an intermediary invests majorly in socially responsible financial products, the level of social responsibility of this intermediary is high correspondingly. Furthermore, it is assumed that for intermediary j, he has a pre-specified minimal social responsibility level, denoted by g_j according to his/her social awareness.

FIGURE 3
Network Structure of Financial Intermediary j's Transactions



Moreover, the intermediaries may have risk associated with transacting with the various source agents and with the demand markets. Let r^j denote the risk function associated with intermediary j's transactions. Assume that it is a strictly convex and continuously differentiable function and that it depends on the financial transactions q_j^2.

The financial intermediary cannot produce more financial products than it has as financial holdings obtained from the various source agents. Therefore, the following constraint holds for each intermediary j:

$$\sum_{k=1}^{o}\sum_{l=1}^{2} q_{jkl}^2 \leq \sum_{i=1}^{m}\sum_{l=1}^{2} q_{ijl}^1 \quad \forall j. \tag{7}$$

Furthermore, each intermediary has his/her pre-specified minimum social responsibility level, which is denoted by g_j. Hence, each intermediary j has the following constraint:

$$\sum_{k=1}^{o}\left(\sum_{l=1}^{2} q_{jkl}^{2} / \sum_{k=1}^{o} \sum_{l=1}^{2} q_{jkl}^{2}\right) I_k \geq g_j \quad \forall j. \tag{8a}$$

After rearranging the terms, (8a) can be put in the following form:

$$\sum_{k=1}^{o} \sum_{l=1}^{2} q_{jkl}^{2} I_k - g_j\left(\sum_{k=1}^{o} \sum_{l=1}^{2} q_{jkl}^{2}\right) \geq 0 \quad \forall j. \tag{8b}$$

In the above constraint, $\sum_{l=1}^{2} q_{jkl}^{2} / \sum_{k=1}^{o} \sum_{l=1}^{2} q_{jkl}^{2}$ is the percentage of the money that intermediary j invested in product k through both physical and electronic transaction modes. With I_k being the social responsibility level of product k, the left hand side of the above constraint is the weighted average of the social responsibility level of intermediary j.

Similarly, each intermediary faces a multi-criteria decision-making problem, with the first objective expressed as:

$$\text{Maximize} \quad z_{1j}^{2} = \sum_{k=1}^{o} \sum_{l=1}^{2}\left(\rho_{jkl}^{2} q_{jkl}^{2} - c_{jkl}(q_{jkl}^{2})\right) - c_j(Q^1) - \sum_{i=1}^{m} \sum_{l=1}^{2}\left(\rho_{ijl}^{1} q_{ijl}^{1} + \hat{c}_{ijl}(q_{ijl})\right) \tag{9}$$

denoting the net revenue to be maximized; the second objective is to maximize the monetary value associated with the social responsibility level expressed as:

$$\text{Maximize} \quad z_{2j}^{2} = SR_j^2(q_j^2) \tag{10}$$

which is assumed to be concave for each j
and the third objective denoting risk minimization expressed as:

$$Minimize z_{3j}^{2} = r^{j}\left(q_j^2\right) \tag{11}$$

In particular, the monetary value of one company's SR is assumed as the incremental value of the shareholder's equity due to the embracement of the SR in the company's culture.

Each intermediary, as the case for each source agent above, is faced with trade-offs among profit, social responsibility level, and risks. Depending on the social awareness of an intermediary, a weight is assigned to reflect his social awareness. Furthermore, to reveal an intermediary's attitude toward risks, a variable weight associated with his risk objective can be constructed in a manner similar to that done above for the source agents. As mentioned earlier, financial intermediaries, relative to individual agents, are able to collect large amount of funds and therefore can build largely diversified portfolios to minimize risks. If total risks can be decomposed into two parts: systematic risk which cannot be eliminated through diversification and diversifiable risk which can be reduced through portfolio diversification, then it is reasonable to believe that individuals and intermediaries shall have the similar exposure to systematic risks; while intermediaries should have advantage reducing much of the diversifiable risk. As a result, it could be that individual agents tend to be more risk averse than financial intermediaries and might penalize risks more than intermediaries do. Assume that the risk-penalizing weight of intermediary j is denoted by $w_{3j}^2\left(r^j\left(q_j^2\right)\right)$ and that is strictly increasing, convex, smooth, and nonnegative for each j (Nagurney and Ke, 2006; Dong and Nagurney, 2001). We further assume that each intermediary will associate a weight with his/her social responsibility level to reflect his/her attitude toward the social responsibility. Unlike the variable weights associated with the risks, which are changed with the risk level, a company's attitude toward social responsibility is determined by the company's social awareness and leadership. Therefore, we set the weights related to the social responsibility level as predetermined parameters.

We then can derive the following risk-penalizing value function of intermediary j:

$$\text{Maximize} \quad U^j\left(q_j^2\right)$$
$$= \sum_{k=1}^{o} \sum_{l=1}^{2} (\rho_{jkl}^2 q_{jkl}^2 - c_{jkl}(q_{jkl}^2)) - c_j(Q^1) - \sum_{i=1}^{m} \sum_{l=1}^{2} (\rho_{ijl}^1 q_{ijl}^1 + \hat{c}_{ijl}(q_{ijl}))$$
$$+ w_{2j}^2 \sum_{j=1}^{n} SR_j^2(q_{jk}^2)$$
$$- w_{3j}^2\left(r^j\left(q_j^2\right)\right) r^j\left(q_j^2\right) \tag{12}$$

Subject to (7) and (8b).

The expression consisting of the first five terms to the right-hand side of (12) represents the net return (to be maximized), the sixth term represents the weighted monetary value of the social responsibility level for intermediary j, whereas the last term in (12) represents the weighted dollar value of risk (to be minimized) associated with the intermediary j's risk attitude. An analogous proof to that of Theorem 1 in Nagurney and Ke (2006) and Qiang et al. (2014) can be applied to establish that U^j is strictly concave with respect to q_j^2 under the above stated assumptions on the transaction cost functions and risk function for intermediary j.

The optimality conditions for all financial intermediaries simultaneously, under the above stated assumptions, can be succinctly expressed as (see also Bertsekas and Tsitsiklis, 1989; Bazaraa et al., 1993; Gabay and Moulin, 1980;Dafermos and Nagurney, 1987; and Nagurney and Ke, 2001; 2003; 2006): Determine $(Q^{1*}, Q^{3*}, \gamma^*, \xi^*) \in R_+^{2mn+2no+n+n}$, such that:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[w_{3j}^2\left(r^j\left(q_j^{2*}\right)\right)\frac{\partial r^j\left(q_j^{2*}\right)}{\partial q_{ijl}^1} + \frac{\partial w_{3j}^2\left(r^j\left(q_j^{2*}\right)\right)}{\partial q_{ijl}^1} r^j\left(q_j^{2*}\right) + \frac{\partial \hat{c}_{ijl}\left(q_{ijl}^{1*}\right)}{\partial q_{ijl}^1} + \right.$$
$$\left.\frac{\partial c_j(Q^{1*})}{\partial q_{ijl}^1} + \rho_{ijl}^{1*} - \gamma_j^*\right] \times \left[q_{ijl}^1 - q_{ijl}^{1*}\right] + \sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[w_{3j}^2\left(r^j\left(q_j^{2*}\right)\right)\frac{\partial r^j\left(q_j^{2*}\right)}{\partial q_{jkl}^2} + \right.$$

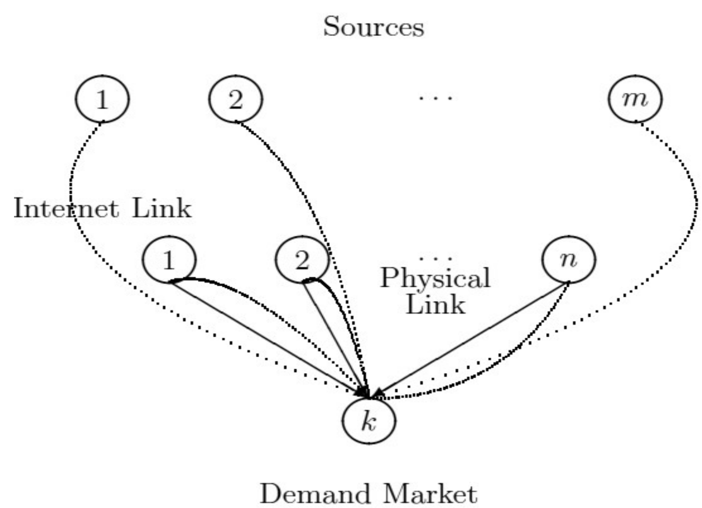
$$\left.\frac{\partial w_{3j}^{2}\left(r^{j}\left(q_{j}^{2*}\right)\right)}{\partial q_{jkl}^{2}} r^{j}\left(q_{j}^{2*}\right) + \frac{\partial c_{jkl}\left(q_{jkl}^{2*}\right)}{\partial q_{jkl}^{2}} - \rho_{jkl}^{2*} + \gamma_{j}^{*} + \left(g_{j} - I_{k}\right)\xi_{j}^{*} - w_{2j}^{2}\frac{\partial SR_{j}^{2}\left(q_{j}^{2*}\right)}{\partial q_{jkl}^{2}}\right] \times$$
$$\left[q_{jkl}^{2} - q_{jkl}^{2*}\right] + \sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{l=1}^{2} q_{ijl}^{1*} - \sum_{k=1}^{o}\sum_{l=1}^{2} q_{jkl}^{2*}\right] \times \left[\gamma_{j} - \gamma_{j}^{*}\right] +$$
$$\sum_{j=1}^{n}\left[\sum_{k=1}^{o} I_{k}\sum_{l=1}^{2} q_{jkl}^{2*} - g_{j}\left(\sum_{k=1}^{o}\sum_{l=1}^{2} q_{jkl}^{2*}\right)\right] \times \left[\xi_{j} - \xi_{j}^{*}\right] \geq$$
$$0, \forall (Q^{1}, Q^{3}, \gamma, \xi) \in R_{+}^{2mn+2no+n+n} \qquad (13)$$

whereγ_j and ξ_j are the Lagrange multipliers associated with constraints (7) and (8b), respectively. (see Bazaraa et al., 1993), γ and ξ are the n-dimensional column vectors of Lagrange multipliers of all the intermediaries, and the top and middle tier prices (without loss of generality) are at their equilibrium values (more discussion on the pricing mechanism follows at the end of this section).

The Consumers at the Demand Markets and the Equilibrium Conditions

The consumers at the demand markets take into account in making their consumption decisions not only the price charged for the financial products but also their transaction costs associated with obtaining the financial products. See Figure 4 for the transactions associated with demand market k.

FIGURE 4
The Network Structure of Transactions at Demand Market k



A demand function d_k is assumed to depend upon the entire vector of prices ρ^3.

$$d_k = d_k(\rho^3) \qquad (14)$$

The equilibrium conditions (with the middle tier prices set at their equilibrium values) for demand market k (cf. Nagurney and Ke, 2001; 2003; 2006), thus, take the form: for all intermediariesjs:

$$\rho_{jkl}^{2*} + \hat{c}_{jkl}(Q^{2*}, Q^{3*}) \begin{cases} = \rho_k^{3*}, \text{ if } q_{jkl}^{2*} > 0 \\ \geq \rho_k^{3*}, \text{ if } q_{jkl}^{2*} = 0 \end{cases} \tag{15}$$

For all source agent i：

$$\rho_{ik}^{1*} + \hat{c}_{ik}(Q^{2*}, Q^{3*}) \begin{cases} = \rho_k^{3*}, \text{ if } q_{ik}^{1*} > 0 \\ \geq \rho_k^{3*}, \text{ if } q_{ik}^{1*} = 0 \end{cases} \tag{16}$$

Condition (15) states that the consumers at demand market k will purchase the financial product from intermediary j if the price charged by the intermediary plus the transaction cost (from the perspective of the consumers) does not exceed the price that the consumers are willing to pay for the product. Same rationale explains condition (16) between demand market and source agent. On the other hand, if the price that the consumers are willing to pay for a financial product is positive, then the quantity of the product at the demand market is precisely equal to the demand. Therefore:

$$d_k(\rho_3^*) \begin{cases} = \sum_{j=1}^{n} \sum_{l=1}^{2} q_{jkl}^{2*} + \sum_{i=1}^{m} q_{ik}^{1*}, \text{ if } \rho_k^{3*} > 0 \\ \leq \sum_{j=1}^{n} \sum_{l=1}^{2} q_{jkl}^{2*} + \sum_{i=1}^{m} q_{ik}^{1*}, \text{ if } \rho_k^{3*} = 0 \end{cases} \tag{17}$$

In equilibrium, conditions (15) and (16) have to hold for all demand markets and these, in turn, can be expressed also as an inequality akin to (6) and (13) and given by:

determine $(Q^{2*}, Q^{3*}, \rho^{3*}) \in R_+^{2no+mo+o}$, such that:

$$\sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{2} \left[\rho_{jkl}^{2*} + \hat{c}_{jkl}\left(Q^{2^*}, Q^{3^*}\right) - \rho_k^{3*}\right] \times \left[q_{jkl}^{2} - q_{jkl}^{2*}\right] + \sum_{i=1}^{m} \sum_{k=1}^{o} \left[\rho_{ik}^{1*} + \hat{c}_{ik}(Q^{2*}, Q^{3*}) - \rho_k^{3*}\right] \times \left[q_{ik}^{1} - q_{ik}^{1*}\right] + \sum_{k=1}^{o} \left[\sum_{j=1}^{n} \sum_{l=1}^{2} q_{jkl}^{2*} + \sum_{i=1}^{m} q_{ik}^{1*} - d_k(\rho_3^*)\right] \times \left[\rho_k^{3} - \rho_k^{3*}\right] \geq 0, \forall (Q^2, Q^3, \rho^3) \in R_+^{2no+mo+o} \tag{18}$$

The Equilibrium Conditions of the Financial Network with Social Responsibility Investing and Electronic Transactions

Combining the above equilibrium conditions (cf. (6), (13), and (18)) of different tiers, we now state:

Definition 1: Financial Network Equilibrium with Social Responsibility Investing and Electronic Transactions

The equilibrium state of the financial network with social responsibility investing and electronic transaction is one where the financial flows between tiers coincide and the financial flows and prices satisfy the sum of conditions (6), (13), and (18).

We now establish the following:

Theorem 1 Variational Inequality Formulation

The equilibrium state governing the financial network with socially responsible investing and variable weights is equivalent to the solution of the variational inequality given by:

determine $(Q^{1*}, Q^{2*}, Q^{3*}, \gamma^*, \xi^*, \rho^{3*}) \in \mathcal{K}$, satisfying

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[w_{2i}^{1}\left(r^{i}\left(q_i^{1*}\right)\right)\frac{\partial r^{i}\left(q_i^{1*}\right)}{\partial q_{ijl}^{1}}+\frac{\partial w_{2i}^{1}\left(r^{i}\left(q_i^{1*}\right)\right)}{\partial q_{ijl}^{1}}r^{i}\left(q_i^{1*}\right)+\right.$$
$$w_{3j}^{2}\left(r^{j}\left(q_j^{2*}\right)\right)\frac{\partial r^{j}\left(q_j^{2*}\right)}{\partial q_{ijl}^{1}}+\frac{\partial w_{3j}^{2}\left(r^{j}\left(q_j^{2*}\right)\right)}{\partial q_{ijl}^{1}}r^{j}\left(q_j^{2*}\right)+\frac{\partial c_{ijl}\left(q_{ijl}^{1*}\right)}{\partial q_{ijl}^{1}}+\frac{\partial \hat{c}_{ijl}\left(q_{ijl}^{1*}\right)}{\partial q_{ijl}^{1}}+$$
$$\left.\frac{\partial c_j(Q^{1*})}{\partial q_{ijl}^{1}}-\gamma_j^*\right]\times\left[q_{ijl}^{1}-q_{ijl}^{1*}\right]+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[w_{3j}^{2}\left(r^{j}\left(q_j^{2*}\right)\right)\frac{\partial r^{j}\left(q_j^{2*}\right)}{\partial q_{jkl}^{2}}+\right.$$
$$\frac{\partial w_{3j}^{2}\left(r^{j}\left(q_j^{2*}\right)\right)}{\partial q_{jkl}^{2}}r^{j}\left(q_j^{2*}\right)+\frac{\partial c_{jkl}\left(q_{jkl}^{2*}\right)}{\partial q_{jkl}^{2}}+\gamma_j^*+\left(g_j-I_k\right)\xi_j^*+\hat{c}_{jk}\left(Q^{2^*}\right)-\rho_k^{3*}-$$
$$\left.w_{2j}^{2}\frac{\partial SR_j^{2}\left(q_j^{2*}\right)}{\partial q_{jkl}^{2}}\right]\times\left[q_{jkl}^{2}-q_{jkl}^{2*}\right]+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[w_{2i}^{1}\left(r^{i}\left(q_i^{1*}\right)\right)\frac{\partial r^{i}\left(q_i^{1*}\right)}{\partial q_{ik}^{1}}+\right.$$
$$\left.\frac{\partial w_{2i}^{1}\left(r^{i}\left(q_i^{1*}\right)\right)}{\partial q_{ik}^{1}}r^{i}\left(q_i^{1*}\right)+\frac{\partial c_{ik}\left(q_{ik}^{1*}\right)}{\partial q_{ik}^{1}}+\hat{c}_{ik}(Q^{2*},Q^{3*})-\rho_k^{3*}\right]\times\left[q_{ik}^{1}-q_{ik}^{1*}\right]+$$
$$\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{l=1}^{2}q_{ijl}^{1*}-\sum_{k=1}^{o}\sum_{l=1}^{2}q_{jkl}^{2*}\right]\times\left[\gamma_j-\gamma_j^*\right]+\sum_{j=1}^{n}\left[\sum_{k=1}^{o}I_k\sum_{l=1}^{2}q_{jkl}^{2*}-\right.$$
$$\left.g_j\left(\sum_{k=1}^{o}\left[\!\left[\sum_{l=1}^{2}q_{jkl}^{2*}\right)\right]\!\right]\right]\times\left[\xi_j-\xi_j^*\right]+\sum_{k=1}^{o}\left[\sum_{j=1}^{n}\sum_{l=1}^{2}q_{jkl}^{2*}+\sum_{i=1}^{m}q_{ik}^{1*}-\right.$$
$$\left.d_k(\rho_3^*)\right]\times\left[\rho_k^3-\rho_k^{3*}\right]\geq 0\qquad\forall(Q^1,Q^2,Q^3,\gamma,\xi,\rho^3)\in K \tag{19}$$

Where $\mathcal{K}\equiv\{K_1\times R_+^{2no+n+n+o}\}$■

The proof is analogous to the proof of Theorem 2 in Nagurney and Ke (2006). For easy reference in the subsequent sections, variational inequality problem (19) can be rewritten in standard variational inequality form (cf. Nagurney, 1999) as follows:

$$F(X^*)^T\cdot(X-X^*)\geq 0 \tag{20}$$

where $X\equiv(Q^1,Q^2,Q^3,\gamma,\xi,\rho^3)$, and $F(X)\equiv(F_{ijl}, F_{jkl}, F_{ik}, F_j, F_j, F_k)_{i=1,\ldots,m;\ j=1,\ldots,n;\ l=1,2;\ k=1,\ldots,o}$ and the specific components of F given by the functional terms preceding the multiplication signs in (19), respectively.

We now discuss how to recover the prices ρ_{ijl}^{1*}, for all i,j,l, and ρ_{jkl}^{2*}, for all j,k,l, from the solution of variational inequality (19). Observe that these prices do not appear in variational inequality (19). However, they do play an important role in terms of pricing of the various financial instruments/products. Note that from (6), if q_{ijl}^{1*}>0, for some i, j, l, then ρ_{ijl}^{1*} is precisely equal to

$w_{2i}^1\left(r^i\left(q_i^{1*}\right)\right)\frac{\partial r^i\left(q_i^{1*}\right)}{\partial q_{ijl}^1}+\frac{\partial w_{2i}^1\left(r^i\left(q_i^{1*}\right)\right)}{\partial q_{ijl}^1}r^i\left(q_i^{1*}\right)+\frac{\partial c_{ijl}\left(q_{ijl}^{1*}\right)}{\partial q_{ijl}^1}$. Similarly, the prices ρ_{jkl}^{2*}, in turn (cf. (13)), can be obtained by finding a $q_{jkl}^{2*}>0$, and then setting $\rho_{jkl}^{2*} = w_{3j}^2\left(r^j\left(q_j^{2*}\right)\right)\frac{\partial r^j\left(q_j^{2*}\right)}{\partial q_{jkl}^2}+\frac{\partial w_{3j}^2\left(r^j\left(q_j^{2*}\right)\right)}{\partial q_{jkl}^2}r^j\left(q_j^{2*}\right)+\frac{\partial c_{jkl}\left(q_{jkl}^{2*}\right)}{\partial q_{jkl}^2}+$ $\gamma_j^*+(g_j-I_k)\xi_j^*-w_{2j}^2\frac{\partial SR_j^2\left(q_{jkl}^{2*}\right)}{\partial q_{jkl}^2}$ for all such j, k, l.

Moreover, it is easy to establish that if the top and middle tier prices are set as above, then the optimality conditions (6) and (13) and the equilibrium conditions (18) each hold (separately).

Hence, using the variational inequality formulation one can not only determine the equilibrium financial flows between the tiers of the financial network but, in addition, the equilibrium prices associated with the financial products at the demand markets, ρ^{3*}, the equilibrium prices at the source agents, ρ^{1*}, and at the intermediaries, ρ^{2*}.

QUALITATIVE PROPERTIES

In this section, we provide some qualitative properties of the solution to the variational inequality (19). In particular, the existence and uniqueness results are derived. We also investigate properties of the function F (cf. (20)) that enters the variational inequality of interest here.

Now we address the following qualitative properties of the financial network model with social responsibilities, electronic transactions and variable weights. Since the feasible set is not compact, we cannot derive existence simply from the assumption of continuity of the functions. Nevertheless, we can impose a rather weak condition to guarantee existence of a solution pattern. Let

$$K_b = \{(Q^1, Q^2, Q^3, \gamma, \xi, \rho^3) | 0 \le Q^1 \le b_1;\ \ 0 \le Q^2 \le b_2; 0 \le Q^3 \le b_3; 0 \le \gamma \le b_4; 0 \le \xi \le b_5; 0 \le \rho^3 \le b_6\},$$

whereb $= (b_1, b_2, b_3, b_4, b_5, b_6) > 0$ and $Q^1 \le b_1;\ \ Q^2 \le b_2; Q^3 \le b_3; \gamma \le b_4; \xi \le b_5; \rho^3 \le b_6$ means that $q_{ijl}^1 \le b_1; q_{jkl}^2 \le b_2; q_{ik}^3 \le b_3; \gamma_j \le b_4; \xi_j \le b_5; \rho_k^3 \le b_6 \quad \forall\ i, j, k, l$. Then K_b is bounded closed convex subset of $R_+^{2mn++mo+2no+n+n+o}$. Thus, the following variational inequality:

$$F\left(X^b\right)^T \cdot \left(X - X^b\right) \ge 0, \qquad \forall\ X \in K_b, \tag{21}$$

admits at least one solution $X_b \in K_b$, from the standard theory of variational inequalities, since K_b is compact and F is continuous. Following Kinderlehrer and Stampacchia (1980) (see also Theorem 1.5 in Nagurney, 1999), we thus have:

Theorem 2

Variational inequality (19) admits a solution if and only if there exists a $b > 0$*, such that variational inequality (20) admits a solution in* K_b *with*
$Q^1 < b_1; \ Q^2 < b_2; Q^3 < b_3; \gamma < b_4; \xi < b_5; \rho^3 < b_6$. (22)

Theorem 3 Existence

Suppose that there exist positive constants M, N, R with R > 0, such that:

$$w_{2i}^1\left(r^i(q_i^1)\right)\frac{\partial r^i(q_i^1)}{\partial q_{ijl}^1}+\frac{\partial w_{2i}^1\left(r^i(q_i^1)\right)}{\partial q_{ijl}^1}r^i\left(q_i^1\right)+w_{3j}^2\left(r^j\left(q_j^2\right)\right)\frac{\partial r^j\left(q_j^2\right)}{\partial q_{ijl}^1}+\frac{\partial w_{3j}^2\left(r^j\left(q_j^2\right)\right)}{\partial q_{ijl}^1}r^j\left(q_j^2\right)+\frac{\partial c_{ijl}\left(q_{ijl}^1\right)}{\partial q_{ijl}^1}+\frac{\partial \hat{c}_{ijl}\left(q_{ijl}^1\right)}{\partial q_{ijl}^1}+\frac{\partial c_j(Q^1)}{\partial q_{ijl}^1}\geq M,\ \forall Q^1\ \ with\ \ \ q_{ijl}^1\geq N,\ \forall i,j,l, \quad (23)$$

$$w_{3j}^2\left(r^j\left(q_j^2\right)\right)\frac{\partial r^j\left(q_j^2\right)}{\partial q_{jkl}^2}+\frac{\partial w_{3j}^2\left(r^j\left(q_j^2\right)\right)}{\partial q_{jkl}^2}r^j\left(q_j^2\right)+\frac{\partial c_{jk}\left(q_{jkl}^2\right)}{\partial q_{jkl}^2}+\left(g_j-I_k\right)\xi_j\ +\hat{c}_{jkl}(Q^2,Q^3)-w_{2j}^2\frac{\partial SR_j^2\left(q_{jkl}^2\right)}{\partial q_{jkl}^2}\geq M,\ \forall Q^3\ \ with\ \ \ q_{jkl}^2\geq N,\ \forall j,k \quad (24)$$

$$w_{2i}^1\left(r^i\left(q_i^{1*}\right)\right)\frac{\partial r^i(q_i^{1*})}{\partial q_{ik}^1}+\frac{\partial w_{2i}^1\left(r^i(q_i^{1*})\right)}{\partial q_{ik}^1}r^i\left(q_i^{1*}\right)+\frac{\partial c_{ik}\left(q_{ik}^{1*}\right)}{\partial q_{ik}^1}+\ \hat{c}_{ik}(Q^{2*},Q^{3*})\geq M,\ \forall Q^2\ \ with\ \ \ q_{ik}^1\geq N,\ \forall i,k \quad (25)$$

$$d_k(\rho_3\ )\leq N,\ \forall\rho_3\qquad with\quad \rho_k^3>R,\forall k \quad (26)$$

Then variational inequality (19); equivalently, variational inequality (20), admits at least one solution.

Proof: Follows using analogous arguments as the proof of existence for Proposition 1 in Nagurney and Zhao (1993).

Assumptions (23) to (26) are economically reasonable, since when the financial flow between a source agent and intermediary is large, we can expect the corresponding sum of the associated marginal costs of handling and transaction from either the source agent's or the intermediary's perspectives to exceed a positive lower bound. Moreover, in the case where the demand price of the financial product as perceived by consumers at a demand market is high, we can expect that the demand for the financial product at the demand market to not exceed a positive bound.

We now establish additional qualitative properties both of the function F that enters the variational inequality problem (cf. (19) and (20)), as well as uniqueness of the equilibrium pattern. Monotonicity and Lipschitz continuity of the function F (under assumptions given below) will be utilized in Section 4 for proving convergence of the algorithmic scheme.

Theorem 4Monotonicity

Assume that the risk functions r^i; i = 1, . . . ,m; and r^j; j = 1, . . . ,n, are strictly convex, that SR_j are concave and that the c_{ijl}, c_{ik} c_j, $\hat{c}_{ijl}$, and c_{jkl} functions are convex; the $\hat{c}_{jkl}$ functions are monotone increasing, and the d_k functions are monotone decreasing functions, for all i, j, k, l. Assume also that the variable weights are all positive. Then the vector function F that enters the variational inequality (20) is monotone, that is,

$$\langle (F(X') - F(X''))^T, (X' - X'') \rangle \geq 0, \qquad \forall\ X', X'' \in K \tag{27}$$

Proof: From the definition of F(X), the left-hand side of inequality (27) is:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\Bigg[(w_{2i}^{1}\left(r^{i}\left(q_{i}^{1\prime}\right)\right)\frac{\partial r^{i}(q_{i}^{1\prime})}{\partial q_{ijl}^{1}} + \frac{\partial w_{2i}^{1}\left(r^{i}(q_{i}^{1\prime})\right)}{\partial q_{ijl}^{1}} r^{i}\left(q_{i}^{1\prime}\right) + w_{3j}^{2}\left(r^{j}\left(q_{j}^{2\prime}\right)\right)\frac{\partial r^{j}\left(q_{j}^{2\prime}\right)}{\partial q_{ijl}^{1}} + \frac{\partial w_{3j}^{2}\left(r^{j}\left(q_{j}^{2\prime}\right)\right)}{\partial q_{ijl}^{1}} r^{j}\left(q_{j}^{2\prime}\right) + \frac{\partial c_{ijl}\left(q_{ijl}^{1\prime}\right)}{\partial q_{ijl}^{1}} + \frac{\partial \hat{c}_{ijl}\left(q_{ijl}^{1\prime}\right)}{\partial q_{ijl}^{1}} + \frac{\partial c_{j}(Q^{1\prime})}{\partial q_{ijl}^{1}} - \gamma_{j}') - (w_{2i}^{1}\left(r^{i}\left(q_{i}^{1''}\right)\right)\frac{\partial r^{i}(q_{i}^{1''})}{\partial q_{ijl}^{1}} + \frac{\partial w_{2i}^{1}\left(r^{i}(q_{i}^{1''})\right)}{\partial q_{ijl}^{1}} r^{i}\left(q_{i}^{1''}\right) + w_{3j}^{2}\left(r^{j}\left(q_{j}^{2''}\right)\right)\frac{\partial r^{j}\left(q_{j}^{2''}\right)}{\partial q_{ijl}^{1}} + \frac{\partial w_{3j}^{2}\left(r^{j}\left(q_{j}^{2''}\right)\right)}{\partial q_{ijl}^{1}} r^{j}\left(q_{j}^{2''}\right) + \frac{\partial c_{ijl}\left(q_{ijl}^{1''}\right)}{\partial q_{ijl}^{1}} + \frac{\partial \hat{c}_{ijl}\left(q_{ijl}^{1''}\right)}{\partial q_{ijl}^{1}} + \frac{\partial c_{j}(Q^{1''})}{\partial q_{ijl}^{1}} - \gamma_{j}'')\Bigg] \times \left[q_{ijl}^{1\prime} - q_{ijl}^{1''}\right] + \sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\Bigg[(w_{3j}^{2}\left(r^{j}\left(q_{j}^{2\prime}\right)\right)\frac{\partial r^{j}\left(q_{j}^{2\prime}\right)}{\partial q_{jkl}^{2}} + \frac{\partial w_{3j}^{2}\left(r^{j}\left(q_{j}^{2\prime}\right)\right)}{\partial q_{jkl}^{2}} r^{j}\left(q_{j}^{2\prime}\right) + \frac{\partial c_{jkl}\left(q_{jkl}^{2}\right)}{\partial q_{jkl}^{2}} + \gamma_{j}' + \left(g_{j} - I_{k}\right)\xi_{j}' + \hat{c}_{jkl}\left(Q^{2\prime}, Q^{3\prime}\right) - \rho_{k}^{3\prime} - w_{2j}^{2}\frac{\partial SR_{j}^{2}\left(q_{jkl}^{2\prime}\right)}{\partial q_{jkl}^{2}}) - (w_{3j}^{2}\left(r^{j}\left(q_{j}^{2''}\right)\right)\frac{\partial r^{j}\left(q_{j}^{2''}\right)}{\partial q_{jkl}^{2}} + \frac{\partial w_{3j}^{2}\left(r^{j}\left(q_{j}^{2''}\right)\right)}{\partial q_{jkl}^{2}} r^{j}\left(q_{j}^{2''}\right) + \frac{\partial c_{jk}\left(q_{jkl}^{2''}\right)}{\partial q_{jkl}^{2}} + \gamma_{j}'' + \left(g_{j} - I_{k}\right)\xi_{j}'' + \hat{c}_{jkl}\left(Q^{2''}, Q^{3''}\right) - \rho_{k}^{3''} - w_{2j}^{2}\frac{\partial SR_{j}^{2}\left(q_{jkl}^{2''}\right)}{\partial q_{jkl}^{2}})\Bigg] \times \left[q_{jkl}^{2\prime} - q_{jkl}^{2''}\right] + \sum_{i=1}^{m}\sum_{k=1}^{o}\Bigg[(w_{2i}^{1}\left(r^{i}\left(q_{i}^{1\prime}\right)\right)\frac{\partial r^{i}(q_{i}^{1\prime})}{\partial q_{ik}^{1}} + \frac{\partial w_{2i}^{1}\left(r^{i}(q_{i}^{1\prime})\right)}{\partial q_{ik}^{1}} r^{i}\left(q_{i}^{1\prime}\right) + \frac{\partial c_{ik}(q_{ik}^{1\prime})}{\partial q_{ik}^{1}} + \hat{c}_{ik}(Q^{2\prime}, Q^{3\prime}) - \rho_{k}^{3\prime}) - (w_{2i}^{1}\left(r^{i}\left(q_{i}^{1''}\right)\right)\frac{\partial r^{i}(q_{i}^{1''})}{\partial q_{ik}^{1}} + \frac{\partial w_{2i}^{1}\left(r^{i}(q_{i}^{1''})\right)}{\partial q_{ik}^{1}} r^{i}\left(q_{i}^{1''}\right) + \frac{\partial c_{ik}(q_{ik}^{1''})}{\partial q_{ik}^{1}} + \hat{c}_{ik}\left(Q^{2''}, Q^{3''}\right) - \rho_{k}^{3''})\Bigg] \times \left[q_{ik}^{1\prime} - q_{ik}^{1''}\right] + \sum_{j=1}^{n}\left[(\sum_{i=1}^{m}\sum_{l=1}^{2} q_{ijl}^{1\prime} - \sum_{k=1}^{o}\sum_{l=1}^{2} q_{jkl}^{2\prime}) - (\sum_{i=1}^{m}\sum_{l=1}^{2} q_{ijl}^{1''} - \sum_{k=1}^{o}\sum_{l=1}^{2} q_{jkl}^{2''})\right] \times \left[\gamma_{j}' - \gamma_{j}''\right] + \sum_{j=1}^{n}\left[(\sum_{k=1}^{o} I_{k}\sum_{l=1}^{2} q_{jkl}^{2\prime} - g_{j}(\sum_{k=1}^{o}[\![\sum_{l=1}^{2} q_{jkl}^{2\prime})]\!]) - (\sum_{k=1}^{o} I_{k}\sum_{l=1}^{2} q_{jkl}^{2''} - g_{j}(\sum_{k=1}^{o}[\![\sum_{l=1}^{2} q_{jkl}^{2''})]\!])\right] \times \left[\xi_{j}' - \xi_{j}''\right] + \sum_{k=1}^{o}\left[\left(\sum_{j=1}^{n}\sum_{l=1}^{2} q_{jkl}^{2\prime} + \sum_{i=1}^{m} q_{ik}^{1\prime} - d_{k}(\rho_{3}')\right) - \left(\sum_{j=1}^{n}\sum_{l=1}^{2} q_{jkl}^{2''} + \sum_{i=1}^{m} q_{ik}^{1''} - d_{k}\left(\rho_{3}''\right)\right)\right] \times \left[\rho_{k}^{3\prime} - \rho_{k}^{3''}\right] \tag{28}$$

After simplifying (28), we obtain:

$$
\begin{aligned}
&\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\Bigg[\Big(w_{2i}^{1}\big(r^{i}(q_i^{1'})\big)\frac{\partial r^{i}(q_i^{1'})}{\partial q_{ijl}^{1}}+\frac{\partial w_{2i}^{1}\big(r^{i}(q_i^{1'})\big)}{\partial q_{ijl}^{1}}r^{i}(q_i^{1'})+\\
&w_{3j}^{2}\big(r^{j}(q_j^{2'})\big)\frac{\partial r^{j}(q_j^{2'})}{\partial q_{ijl}^{1}}+\frac{\partial w_{3j}^{2}\big(r^{j}(q_j^{2'})\big)}{\partial q_{ijl}^{1}}r^{j}(q_j^{2'})+\frac{\partial c_{ijl}(q_{ijl}^{1'})}{\partial q_{ijl}^{1}}+\frac{\partial \hat{c}_{ijl}(q_{ijl}^{1'})}{\partial q_{ijl}^{1}}+\\
&\frac{\partial c_j(Q^{1'})}{\partial q_{ijl}^{1}}\Big)-\Big(w_{2i}^{1}\big(r^{i}(q_i^{1''})\big)\frac{\partial r^{i}(q_i^{1''})}{\partial q_{ijl}^{1}}+\frac{\partial w_{2i}^{1}\big(r^{i}(q_i^{1''})\big)}{\partial q_{ijl}^{1}}r^{i}(q_i^{1''})+\\
&w_{3j}^{2}\big(r^{j}(q_j^{2''})\big)\frac{\partial r^{j}(q_j^{2''})}{\partial q_{ijl}^{1}}+\frac{\partial w_{3j}^{2}\big(r^{j}(q_j^{2''})\big)}{\partial q_{ijl}^{1}}r^{j}(q_j^{2''})+\frac{\partial c_{ijl}(q_{ijl}^{1''})}{\partial q_{ijl}^{1}}+\frac{\partial \hat{c}_{ijl}(q_{ijl}^{1''})}{\partial q_{ijl}^{1}}+\\
&\frac{\partial c_j(Q^{1''})}{\partial q_{ijl}^{1}}\Big)\Bigg]\times\left[q_{ijl}^{1'}-q_{ijl}^{1''}\right]+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\Bigg[\Big(w_{3j}^{2}\big(r^{j}(q_j^{2'})\big)\frac{\partial r^{j}(q_j^{2'})}{\partial q_{jkl}^{2}}+\\
&\frac{\partial w_{3j}^{2}\big(r^{j}(q_j^{2'})\big)}{\partial q_{jkl}^{2}}r^{j}(q_j^{2'})+\frac{\partial c_{jkl}(q_{jkl}^{2'})}{\partial q_{jkl}^{2}}+\hat{c}_{jkl}(Q^{2'},Q^{3'})-\rho_k^{3'}-w_{2j}^{2}\frac{\partial SR_j^{2}(q_{jkl}^{2'})}{\partial q_{jkl}^{2}}\Big)-\\
&\Big(w_{3j}^{2}\big(r^{j}(q_j^{2''})\big)\frac{\partial r^{j}(q_j^{2''})}{\partial q_{jkl}^{2}}+\frac{\partial w_{3j}^{2}\big(r^{j}(q_j^{2''})\big)}{\partial q_{jkl}^{2}}r^{j}(q_j^{2''})+\frac{\partial c_{jk}(q_{jkl}^{2''})}{\partial q_{jkl}^{2}}+\hat{c}_{jkl}\left(Q^{2''},Q^{3''}\right)-\\
&\rho_k^{3''}-w_{2j}^{2}\frac{\partial SR_j^{2}(q_{jkl}^{2''})}{\partial q_{jkl}^{2}}\Big)\Bigg]\times\left[q_{jkl}^{2'}-q_{jkl}^{2''}\right]+\sum_{i=1}^{m}\sum_{k=1}^{o}\Bigg[\Big(w_{2i}^{1}\big(r^{i}(q_i^{1'})\big)\frac{\partial r^{i}(q_i^{1'})}{\partial q_{ik}^{1}}+\\
&\frac{\partial w_{2i}^{1}\big(r^{i}(q_i^{1'})\big)}{\partial q_{ik}^{1}}r^{i}(q_i^{1'})+\frac{\partial c_{ik}(q_{ik}^{1'})}{\partial q_{ik}^{1}}+\hat{c}_{ik}(Q^{2'},Q^{3'})-\rho_k^{3'}\Big)-\\
&\Big(w_{2i}^{1}\big(r^{i}(q_i^{1''})\big)\frac{\partial r^{i}(q_i^{1''})}{\partial q_{ik}^{1}}+\frac{\partial w_{2i}^{1}\big(r^{i}(q_i^{1''})\big)}{\partial q_{ik}^{1}}r^{i}(q_i^{1''})+\frac{\partial c_{ik}(q_{ik}^{1''})}{\partial q_{ik}^{1}}+\hat{c}_{ik}(Q^{2''},Q^{3''})-\\
&\rho_k^{3''}\Big)\Bigg]\times\left[q_{ik}^{1'}-q_{ik}^{1''}\right]+\sum_{k=1}^{o}\Big[\Big(\sum_{j=1}^{n}\sum_{l=1}^{2}q_{jkl}^{2'}+\sum_{i=1}^{m}q_{ik}^{1'}-d_k(\rho_3')\Big)-\\
&\Big(\sum_{j=1}^{n}\sum_{l=1}^{2}q_{jkl}^{2''}+\sum_{i=1}^{m}q_{ik}^{1''}-d_k(\rho_3^{''})\Big)\Big]\times\left[\rho_k^{3'}-\rho_k^{3''}\right] \qquad (29)
\end{aligned}
$$

It is easy to verify that under the above imposed assumptions the term in (29) is greater than or equal to zero. ■

Monotonicity plays a role in the qualitative analysis of variational inequality problems similar to that played by convexity in the context of optimization problems.

Since the proof of Theorems 5 below is similar to that of Theorem 4, it is omitted here.

Theorem 5: Strict Monotonicity

Assume all the conditions of Theorem 4. In addition, suppose that one of the families of convex functions c_{ijl}; *i=1,…,m; j=1,…,n; l=1,…,L;* c_j; *j=1,…,n;* $\hat{c}_{ijl}$; *i=1,…,m; j=1,…,n; l=1,…,L; and* c_{jk}; *j=1,…,n; k=1,…,o, is a family of strictly convex functions. Suppose also that* $\hat{c}_{jk}$; *j=1,…,n; k=1,…,o, functions and the -*d_k; *k=1,…,o, functions are strictly monotone. Then, the vector function F that enters the variational inequality(20) is strictly* *monotone, with respect to* (Q^1, Q^2, ρ^3)*, that is, for any two* $X', X" \in K$ *with* $(Q^{1\prime}, Q^{2\prime}, \rho^{3\prime}) \neq (Q^{1\prime\prime}, Q^{2\prime\prime}, \rho^{3\prime\prime})$

$$\langle (F(X') - F(X"))^T, (X' - X") \rangle > 0, \qquad \forall \; X', X" \in K \quad , \tag{30}$$

Theorem 6: Uniqueness

Assuming the conditions of Theorem 5, there must be a unique financial flow pattern (Q^{1*}, Q^{2*}, Q^{3*})*, and a unique demand price vector* ρ^{3*} *satisfying the equilibrium conditions of the financial network with socially responsible investing. In other words, if the variational inequality (19) admits a solution, then that is the only solution in* $(Q^1, Q^2, Q^{3*}, \rho^3)$*.*

CONCLUSION AND FUTURE RESEARCH

In this paper, we proposed a financial network equilibrium model with socially responsible intermediaries and variable weights. The financial network consists of a multi-tiered network that is formed by source agents, the financial intermediaries, and the consumers associated with the financial products at the demand markets. Each decision-maker in the first two tiers of the network faces multi-criteria decision-making problem, including profit maximization and risk minimization. Furthermore, similar to the work of Nagurney and Ke (2006) and Qiang et al. (2013), variable weights are associated with the risk functions to reflect the risk attitude of the decision-makers. In addition, intermediaries are assumed to be socially responsible financial institutes, who have an additional objective to maximize their social responsible level. We derived the optimality conditions for the source agents as well as the financial intermediaries under appropriate assumptions on the functions. Equilibrium conditions are provided and then the corresponding governing equivalent variational inequality formulation is derived. Subsequently, after applying the modified projection algorithm, an illustration example is given to provide managerial insights on the impact of the social responsibility on the financial flows in the network.

To extend this research in the future, we would like to capture the heterogeneity among the investors on their social awareness. Also, collecting data to empirically test our model is also desirable.

REFERENCE

Ang, W.R., Gregoriou, G.N., and Lean, H.H. 2014. Market-timing skills of socially responsible investment fund managers: The case of North America versus Europe. *Journal of Asset Management* (15:6), 366-377.

Auer, B.R. 2016. Do Socially Responsible Investment Policies Add or Destroy European Stock Portfolio Value? *Journal of Business Ethics* (135:2), 381-397.

Ballestero, E., Pérez-Gladish, B., and Garcia-Bernabeu, A. 2015. Socially responsible investment. *A multi-criteria decision making approach*

(International series in operations research & management science, Vol. 219), Springer, Cham.

Basso, A., and Funari, S. 2014. Constant and variable returns to scale DEA models for socially responsible investment funds. *European Journal of Operational Research* (235:3), 775-783.

Bazaraa, M.S., Sherali, H.D., and Shetty, C.M. 1993. *Nonlinear programming: Theory and algorithms*, John Wiley & Sons, New York.

Bertsekas, D.P., and Tsitsiklis, J.N. 1989. *Parallel and distributed computation - numerical methods*, Prentice Hall, Englewood Cliffs, New Jersey.

Boutin-Dufresne, F., and Savaria, P. 2004. Corporate social responsibility and financial risk. *Journal of Business Ethics* (76:2), 189-206.

Bilbao-Terol, A., Arenas-Parra, M., Cañal-Fernández, V., and Bilbao-Terol, C. 2015. Multi-criteria decision making for choosing socially responsible investment within a behavioral portfolio theory framework: a new way of investing into a crisis environment. *Annals of Operations Research*, 1-32.

Celent 2007. Socially responsible investing in the US and Europe: same goals but different paths. (http://www.celent.com/reports/socially-responsible-investing-us-and-europe-same-goals-different-paths; accessed on May 2nd, 2016).

Chankong, V., and Haimes, Y.Y. 1983. *Multiobjective decision making: Theory and methodology*, North-Holland, New York.

Dafermos, S., and Nagurney, A. 1987. Oligopolistic and competitive behavior of spatially separated markets. *Regional Science and Urban Economics* (17:2), 245-254.

Daniele, P., Giuffré, S., and Pia, S. 2005. Competitive financial equilibrium problems with policy interventions. *Journal of Industrial and Management Optimization* (1:1), 39-52.

Dong, J., and Nagurney, A. 2001. Bicriteria decision making and financial equilibrium: a variational inequality perspective. *Computational Economics* (17:1), 29-42.

Fishburn, P.C. 1970. *Utility theory for decision making*, John Wiley & Sons, New York.

Folwer, S.J., and Hope, C. 2007. A critical review of sustainable business indices and their Impact. *Journal of Business Ethics* (76:3), 243-252

Gabay, D., and Moulin, H. 1980. On the uniqueness and stability of Nash equilibria in Noncooperative games. In: A. Bensoussan, P. Kleindorfer, and C.S. Tapiero (ed.), *Applied stochastic control of econometrics and management science*, North-Holland, Amsterdam, The Netherlands, 271-294.

Géczy, C., Stambaugh, R.F., and Levin, D. 2003. Investing in socially responsible mutual funds. Working paper, Wharton School, *University of Pennsylvania*.

Giuffrè, S., and Pia, S. 2005. Variational inequalities for time dependent financial equilibrium with price constraints. In: F. Giannessi, and A. Maugeri (ed.) *Analysis and applications, Series: nonconvex optimization and its applications*, Springer, New York.

Guenes, J., and Pardalos, P.M. 2003. Network optimization in supply chain management and financial engineering: an annotated bibliography. *Networks* (42:2), 66-84.
Guerad, J.B. Jr. 1997. Is there a cost to being socially responsible in investing? *Journal of Investing* (6:2), 11-18.
Hickman, K.A., Teets, W.R., and Kohls, J.J. 1999. Social investing and modern portfolio theory. *American Business Review* (17:1), 72-78.
Humphrey, J. 2015. Socially responsible screening in mutual funds. In T. Hebb, J. Hawley, A. Hoepner, A. Neher, and D. Wood (ed.), *The Routledge Handbook of Responsible Investment*, Routledge, Abingdon, Oxon, United Kingdom, 667-676.
Jun, H. 2016. Corporate governance and the institutionalization of socially responsible investing (SRI) in Korea. *Asia Pacific Business Review* (22:3), 487-501.
Keeney, R.L., and Raiffa, H. 1992. *Decisions with multiple objectives: preferences and value tradeoffs*, Cambridge University Press, Cambridge, England.
Kinderlehrer, D., Stampacchia, G. 1980. *An introduction to variational inequalities and their application*, Academic Press, New York.
Krumsiek, B.J. 1997. The emergence of a new era in mutual fund investing socially responsible investing comes of age. *Journal of Investing* (6:4), 25-30.
Kurtz, L. 1997. No effect, or no net effect? studies on socially responsible investing. *Journal of Investing* (6:4), 37-49.
Markowitz, H.M. 1952. Portfolio selection. *Journal of Finance* (7:1), 77-91.
McWilliams, A., and Siegel, D. 1997. Events studies in management research: theoretical and empirical issues. *Academy of Management Journal* (40:3), 568-592.
Nagurney, A. 1999. *Network economics: A variational inequality approach (2nd and revised ed.)*, Kluwer Academic Publishers, Dordrecht, The Netherlands.
Nagurney, A. 2008. Networks in finance. In: D. Seese, C. Weinhardt, and F. Schlottmann (ed.), *The handbook on information technology and finance*, Springer, Berlin, Germany, 383-420.
Nagurney, A., and Ke, K. 2001. Financial networks with intermediation. *Quantitative Finance* (1:4), 441-451.
Nagurney, A, and Ke, K. 2003. Financial networks with electronic transactions: modeling, analysis, and computations. *Quantitative Finance* (3:2), 71-87.
Nagurney, A., and Ke, K. 2006. Financial networks with intermediation: risk management with variable weights. *European Journal of Operational Research* (172:1), 40-63.
Nagurney, A., and Zhao, L. 1993. Networks and variational inequalities in the formulation and computation of market disequilibria: the case of direct demand functions. *Transportation Science* (27:1), 4-15.
Pava, M.L., and Krausz, J. 1996. The association between corporate social-responsibility and financial performance: The paradox of social cost. *Journal of Business Ethics* (15:3), 321-357.

Porter, M.E., and Kramer, M.R.2006. The link between competitive advantage and corporate social responsibility. *Harvard Business Review* (84:12), 78-92.
Preston, L.E., and O'Bannon, D.P. 1997. The corporate social-financial performance relationship: a typology and analysis. *Business Society* (36:4), 419-429.
Qiang, Q., Ke, K., and Hu, Y. 2013. Financial networks with socially responsible investing. *Computational Management Science* (10:2-3), 231-252.
Renneboog, L., Ter Horst, J., and Zhang, C. 2011. Is ethical money financially smart? Nonfinancial attributes and money flows of socially responsible investment funds. *Journal of Financial Intermediation* (20:4), 562-588.
Reputation Institute 2010. Global reputation pulse 2010: top line report of the most reputable companies in the world.
Social Investment Forum Foundation 2010. http://ussif.org/news/releases/pressrelease.cfm?id=168
Stanwick, P.A. and Stanwick, S.D. 1998. The relationship between corporate social performance, and organizational size, financial performance, and environmental performance: an empirical examination. *Journal of Business Ethics* (17:2), 195-204.
Statman, M. 2006. Socially responsible indexes: composition, performance, and tracking error. *Journal of Portfolio Management* (32:3), 100-109.
Steuer, R.E., and Na, P. 2003. Multiple criteria decision making combined with finance: A categorized bibliographic study. *European Journal of Operational Research* (150:3), 496-515.
Van de Velde, E., Vermeir, W., and Corten, F. 2005. Corporate social responsibility and financial performance. *Corporate Governance* (5:3), 129-138.
Yu, G.Y. 1997. A multiple criteria approach to choosing an efficient stock portfolio at the Helsinki stock exchange. *Journal of Euro-Asian Management* (3:2), 53-85.
Zeleny, M. 1982. *Multiple criteria decision making*, McGraw-Hill, New York.

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